Multiscale modeling of sea clutter

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Outline

- Background and challenges
- Fitting nonstationary sea clutter data
- Target detection by fully characterizing the correlation structure of sea clutter
- Target detection by cascade multifractal modeling of sea clutter
- Conclusions
Sea clutter

Backscattered returns from a patch of the sea surface illuminated by a transmitted radar pulse

Complexities: turbulent wave motions + multipath propagation
Source of sea clutter data

- 14 sea clutter measurements from Prof. Simon Haykin; each measurement contains 14 range bins, a few bins hit a small target.
- Each measurement was made under certain weather and sea conditions (wave height varied from 0.8 m to 3.8 m; wind conditions varied from still to 60 km/hr).

\[ B_1 \sim B_{14} : \text{Range bins} \]

- \( h \): Antenna height
- \( \phi \): Grazing angle
- \( R_i \): Range (distance from the radar)

\[ B_1 \sim B_{14} : \text{Range bins} \]

\[ R_i \]

\( (\text{Secondary})(\text{Primary})(\text{Secondary}) \)
Significance and Challenges of sea clutter modeling

- Sea clutter analysis is an important theoretical problem
- Target detection within sea clutter is important to coastal and national security, to navigation safety, and to environmental monitoring
- **CouldSat**: Sea clutter removal may help improve cloud system modeling
- Over a thousand papers have been published. Numerous methods and new concepts including chaos and fractal theory have been tried to model sea clutter
- By now, the nature of sea clutter is still not well understood
- Simple and effective models for sea clutter are highly desirable
Why sea clutter modeling is difficult? — Nonstationary!

(i) Data viewed at different time and scales appear very different.

(ii) Subplot (e): signals cannot be characterized as ideal random fractals or autoregressive (AR) processes.
Failure of direct distributional analysis of sea clutter

(i) Distr. tried: Weibull, log-normal, K, compound Gaussian, log-Weibull

(ii) K disrt. \( f(x) = \frac{\sqrt{2\nu}}{\mu \Gamma(\nu)} \left( \sqrt{\frac{2\nu}{\mu}} x \right)^\nu K_{\nu-1} \left( \sqrt{\frac{2\nu}{\mu}} x \right), \quad x \geq 0 \)

is among the best; but the fitting can be poor

— Can’t help with target detection — Culprit: data is nonstationary!
Our approach to fit nonstationary sea clutter data

• Denote the sea clutter amplitude data by \( y(n), n = 1, 2, \cdots \)

• Denote the differenced data of sea clutter by,
  \[ x(n) = y(n + 1) - y(n), \quad n = 1, 2, \cdots \]

• Fit \( x(n), n = 1, 2, \cdots \) using Tsallis distribution.

• Why such a strategy works?
  – Consider white Gaussian noise, \( u(i), i = 1, 2, \cdots \). It is stationary!
  – Standard Brownian motion (or random walk): \( v(n) = \sum_{i=1}^{n} u(i) \) is nonstationary, because the variance of \( v(n) \) is proportional to (time) \( n \)
  – K-distr. can be derived by assuming a random walk model for scatterers!
Tsallis distribution

- Obtained by maximizing the Tsallis entropy under 2 constraints.
- The distr: when $1 < q < 3$,

$$p(x) = \frac{1}{Z_q} [1 + \beta(q - 1)x^2]^{1/(1-q)},$$

where $Z_q$ is a normalization constant.
- When $q = 1$ & $2$, it reduces to the normal & Cauchy distr.
- When $5/3 < q < 3$, the distribution is heavy-tailed
- Significance: provides foundation for the heavy-tailed and $\alpha$-stable distr.
Heavy-tailed distribution

• Pareto distr: $P[X \geq x] = \left(\frac{b}{x}\right)^\alpha$, $x \geq b > 0$, $\alpha > 0$
  where $\alpha$ and $b$ are the shape & the location parameters.

• In the discrete time case, we have Zipf distr.

• Heavy-tailed distr: $P[X \geq x] \sim x^{-\alpha}$, $x \to \infty$

• When $\alpha < 2$, the variance and all higher than 2nd-order moments do not exist.
  when $\alpha \leq 1$, the mean also diverges.

• Cauchy distr (also called Lorentzian distr) with PDF $f(x) = \frac{1}{\pi(l^2+x^2)}$
  is an example with $\alpha = 1$
Stable laws and Levy motions

- Paul Levy (teacher of Mandelbrot, the Father of fractal geometry) posed such a question: When will the distribution for the sum of the random variables and those being summed have the same functional form?

- Stable laws are the unique class of distributions that have such a property.

- Stable laws include Gaussian distr as a special case; in the non-Gaussian case, the distributions are heavy-tailed

- Levy motions: random walk processes whose increments are characterized by stable laws
The meaning of stable laws and Levy motions

• Normal distr & central limit theorem describe daily, mundane life — Many lucky people live through such a life happily.

• Occasionally one has to take on an unplanned journey, during which many unexpected and exciting (or terrible) things happen.

• Such a journey could be related to hate, love, patriotism, and so on, as illustrated by numerous classic poems, fictions and movies.

• **Kolmogorov was pondering**: Stable laws with infinite variance should be observed more often than the normal distr. In reality ...?

• Abundant examples of heavy-tailed distributions have been found: Amount of Internet traffic, topology of networks (eg, power-law networks), distr. of the size of the power outages, ...

• Fundamental question: How do stable laws arise?
Deriving Tsallis distr by maximizing Tsallis entropy

- Tsallis entropy aims to characterize a type of motion whose complexity is neither regular nor fully chaotic/random, by employing a parameter $q$, that best describes the motion.

- It’s defined by 

  $$H_q^T = \frac{1}{q-1} \left( 1 - \sum_{i=1}^{m} p_i^q \right).$$

- In the continuous case, it is 

  $$H_q^T = \frac{1}{q-1} \left( 1 - \int_{-\infty}^{\infty} d\left( \frac{x}{\sigma} \right) [\sigma p(x)]^q \right).$$

- It reduces to the Shannon entropy when $q \to 1$.

- Tsallis distr can be derived by maximizing Tsallis entropy under 2 constraints,
  - Total prob. is 1: $\int_{-\infty}^{\infty} p(x) dx = 1$.
  - Second normalized moment is known: $\int_{-\infty}^{\infty} [x^2 - \sigma^2] [p(x)]^q dx = 0$. 
Generalized Tsallis distribution

- We may generalize the Tsallis distr by replacing the 2nd constraint by \( \int_{-\infty}^{\infty} [x^\alpha - \sigma^\alpha][p(x)]^q dx = 0 \). Then the distr becomes

\[
p(x) = \frac{1}{Z_q} [1 + \beta(q - 1)x^\alpha]^{1/(1-q)}
\]

- This is our starting point for modeling sea clutter.

- To model turbulent motions, Christian Beck (2000) obtained the same distr. through a different approach, which is considerably more complicated than our approach.
Fitting sea clutter by Tsallis distribution

(Symbol: data; curve: Tsallis fit)

- Data is ready. So what is the challenge here?
- Data is highly nonstationary. It’s not very meaningful to perform distributional analysis on original data.
- How about the differenced data? It works!
- Parameters are helpful for target detection.
Introduction to fractal & multifractal

- A part is (exactly or statistically) similar to another part, or to the whole.
- Clouds; mountains; trees; etc. (Images: not computer-made, but photos of Jiu Zhai Gou)
- Power-law relation — a straight line in a log-log plot (scaling)
- Many (or possibly infinitely many) power-law relations — Multifractal.
The set consists of $\infty$ of isolated points. Its measure and topological dimension are both 0. Fractal dimension $= \frac{\ln 2}{\ln 3}$.
Fractional Brownian motion (fBm) $B_H(t)$

- Gaussian process with mean 0 & stationary increments
- Variance:
  \[ E[(B_H(t))^2] = t^{2H} \]
- Power spectral density
  \[ f^{-(2H+1)} \]
- $H$: Hurst parameter.
  - $1/2 < H < 1$: long memory (long-range-dependence (LRD))
  - $H = 1/2$: standard Brownian motion
  - $0 < H < 1/2$: anti-persistence
- Applications to a wide range of problems (including Hollywood movie making—fancy landscape)
Examples of fBm processes with different $H$

(a) $H=0.25$

(b) $H=0.50$

(c) $H=0.75$

(d) $H=0.90$
Random walks and their analysis

- Remove the mean values from \( \{x(i)\} \) process, denote it as \( \{u(i)\} \)
- Random walk: \( y(n) = \sum_{i=1}^{n} u(i) \)
- Independent \( u(i) \)'s (a drunk)—no correlation:
  \[
  E[y(m)^2] = m \cdot E[u(i)^2] \sim m
  \]
- Fluctuation analysis (FA):
  \[
  F^{(2)}(m) = \langle |y(n+m) - y(n)|^2 \rangle \sim m\xi(2)
  \]
  Hurst parameter \( H = H(2) = \xi(2)/2 \)
  - \( H = 1/2 \): no or short-range correlation
  - \( 0 < H < 1/2 \): anti-persistent long range correlation
  - \( 1/2 < H < 1 \): persistent long range correlation
The meaning of the Hurst parameter

- Increment process \( \{x_1, x_2, \cdots, x_n\} \): power spectral density (PSD) \( f^{-(2H-1)} \); autocorrelation function: \( r(k) \sim k^{2H-2}, \) as \( k \to \infty \)
- Random walk process \( \{y_n\}, y_n = \sum_{i=1}^{n} x_i \), PSD: \( f^{-(2H+1)} \)
- Averaging the original series \( X \) over non-overlapping blocks of size \( m \) to obtain:
  \[
  X_t^{(m)} = (X_{tm-m+1} + \cdots + X_{tm})/m, \quad t \geq 1, \quad \text{var}(X^{(m)}) = \sigma^2 m^{2H-2}
  \]
  where \( \sigma^2 \) is the variance of \( \{x_1, x_2, \cdots, x_n\} \)
- The value of \( H \) determines effectiveness of smoothing:
  - \( H = 0.50, m = 100, \text{var}(X^{(m)}) = \sigma^2 / 100 \)
  - \( H = 0.75, m = 10^4, \text{var}(X^{(m)}) = \sigma^2 / 100 \)
  - \( H = 0.25, m \approx 21.5, \text{var}(X^{(m)}) = \sigma^2 / 100 \)
Structure-function–based multifractal analysis

- \( F^{(q)}(m) = \langle |y(i + m) - y(i)|^q \rangle \sim m^{\xi(q)} \)?
  - \( q < 0 \): emphasizes small absolute increments of \( y(i) \);
  - \( q > 0 \): emphasizes large absolute increments of \( y(i) \)

- \( H(q) = \xi(q)/q \)

- Monofractal: \( \xi(q) \) linear in \( q \) (\( \xi(0) = 0 \));
  - \( H(q) \) constant

  Multifractal: \( \xi(q) \) nonlinear in \( q \);
  - \( H(q) \) varies with \( q \)

- Can extend to detrended multifractal and wavelet-based multifractal analysis
  — When analyzing real data, these are preferred!

(Gao et al., *Phys. Rev. E* 2006)
Target detection within sea clutter

- $H(2)$ is much larger when the range bins hit a target
- Sea clutter data are multifractals, and that other $q$ values can also robustly detect targets within sea clutter
Accuracy of target detection across measurements

- Hypothesis $H_0$: sea clutter without target, $H(2) < \gamma$
- Hypothesis $H_1$: sea clutter with target, $H(2) > \gamma$
- $\gamma \approx 0.185$ yields a perfect classification for all datasets

![Graph showing frequency of targets and primary targets across $H$ values]
Modeling multifractals: Cantor set with multifractal measure

$w, w', r, r', 1 - r, 1 - r'$: governed by the same pdf $P(r)$. 
Cascade multifractals: construction rule

<table>
<thead>
<tr>
<th>Stage</th>
<th>Time scale</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>$\varepsilon = 2^0$</td>
</tr>
<tr>
<td>1</td>
<td>$\varepsilon = 2^{-1}$</td>
</tr>
<tr>
<td>2</td>
<td>$\varepsilon = 2^{-2}$</td>
</tr>
<tr>
<td>3</td>
<td>$\varepsilon = 2^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon = 2^{-3}$</td>
</tr>
</tbody>
</table>

All $r_{l,m}, 1 - r_{l,m}$ are governed by same pdf $P(r)$. 
Multifractal scalings for cascade models

- The weights at the stage $N$, $\{w_n, n = 1, \ldots, 2^N\}$, can be expressed as $w_n = u_1 u_2 \cdots u_N$, where $u_l, l = 1, \ldots, N$, are either $r_{ij}$ or $1 - r_{ij}$.
- Thus, $\{u_i, i \geq 1\}$ are independent identically distributed (iid) random variables (RV’s) having pdf $P(r)$.
- Since $\ln w_n$ is the sum of iid RV’s $\ln u_i, i = 1, \ldots, N$, one readily sees that $\ln w_n$ follows a normal distribution, and thus $w_n$ follows a log-normal distribution.
- Multifractal scaling for the cascade model

$$M_q(\varepsilon) = \sum_i w_i^q \sim \varepsilon^{\tau(q)}, \quad D_q = \tau(q)/(q - 1)$$

- We can also prove that

$$\tau(q) = qH(q) - 1$$
Stage-dependent multiplicative process model

Variance of $P(i,i+1)(r)$ varies from one stage to the next in a simple manner:

$$\sigma^2_{(i,i+1)} = a \cdot \sigma^2_{(i-1,i)}, \quad a > 1$$
Sea clutter amplitude and envelope data

Envelopes are formed by picking up successive local maxima.
Multifractal features of sea clutter (Gao & Yao)

Original signal: scaling breaks for negative $q$ and small time scale; indicating the smooth waveform between successive maxima does not follow the multifractal scaling law.

![Graphs showing multifractal features](image-url)
Log-normality of sea clutter envelope signals (Gao & Yao)

Original signal: slightly deviates from log-normal distribution — due to the smooth waveform part.

Envelope signal: excellent log-normal distribution.
Cascade multifractal modeling of sea clutter

- (a,b) Sea clutter amplitude data without and with target.
- (c,d) The corresponding simulated data.
Target detection by cascade multifractal modeling

(a) (b)

Primary
Secondary
Non target

Primary
Secondary
Non target

$D_q$ vs. $q$
Conclusions

- We have shown that sea clutter data are highly nonstationary and multiscaled
- We have developed new distributional analyses approaches to better describe sea clutter
- We have developed structure-function based highly accurate (close to 100%) multifractal methods for detecting low observable targets within sea clutter
- We have developed a cascade multifractal model for sea clutter, which can simultaneously account for the distributional as well as correlation structure of sea clutter
- For more details on the theory, see Gao et al. 
Some thoughts on reducing sea clutter from CoudSat data

- Extend the 1-D cascade multifractal model to 2-D and 3-D (after each partition, one square becomes 4 squares, and one cube becomes 8 cubes)
- Identify important spatial scales associated with wave and turbulence patterns on the sea surface; these scales are important elements in multifractal modeling
- Estimate the Hurst parameter (and the $H(q)$ spectrum) from spatial sea clutter data; they may be of critical importance in designing the best spatial smoothing algorithms
- Non-Gaussian sea clutter distribution may also be exploited to improve spatial smoothing